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SELF CENTEREDNESS IN INTUITIONISTIC FUZZY GRAPH STRUCTURE

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Abstract

In this paper, we studied the self centered property of intuitionistic fuzzy graph structure. It is shown that every complete intuitionistic fuzzy graph structure is self centered but converse is not true. We also derived a necessary and sufficient condition for the intuitionistic fuzzy graph structure to be self centered. An embedding of the center of intuitionistic fuzzy graph structure into a connected self centered intuitionistic fuzzy graph structure is established.

Key words: Intuitionistic fuzzy graph structure, self centre, radius, diameter, edge, path.

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1. Introduction:

The notion of self centered graph was primarily studied by Buckley in [3] and that of almost self centered graph was studied by Klavzar, Narayankar and Walikar in [9]. The set of vertices with minimum eccentricity is termed as a center of a graph. The center of a graph is one of the central concepts in location theory. A graph in which any of its vertices is central is called a centered graph. So graphs in which all vertices are central are called self-centered (SC) graphs. The intuitionistic fuzzification of the concept of self centered graph is

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studied by Karunambigai and Kalaivani in [5]. The notion of intuitionistic fuzzy graph structure (IFGS) is defined and discussed by the authors in [6]. Some elementary operations on IFGSs are discussed in [7]. In this paper, we study the self-centered property of IFGS. The terms like B_i-length, B_i- radius, B_i- diameter etc. in IFGS are introduced. Some results on self centeredness in IFGS are also obtained. Embedding of a self centered IFGS is studied. It is shown that center of IFGS can be embedded into connected self centered IFGS.

2. Preliminaries: In this section, we review some definitions that are necessary in this paper and these are mainly taken from [2], [5], [6] and [8].

Definition (2.1): A simple graph G = (V, E) is a pair of vertex set (V) and an edge set (E) where $E \subseteq V \times V$ i.e., E is a relation on V.

Definition (2.2): The eccentricity $e_G(v)$ or e(v) of a vertex v in a graph G is the distance to a farthest vertex from v. i.e., $e_G(v) = \max\{ d_G(u, v) : \forall u \in V\}$ and the set $\{ e_G(v) : \forall v \in V\}$is called the eccentric set of G.

Definition (2.3): The radius r(G) of G and the diameter d(G) of G are the minimum and the maximum eccentricity, respectively. i.e., $r(G) = \min\{ e_G(v) : \forall v \in V\}$ and $d(G) = \max\{ e_G(v) : \forall v \in V\}$.

Definition (2.4): The center C(G) of a graph G is the set of vertices with minimum eccentricity i.e., the set $C(G) = \{ v \in V : e_G(v) = r(G) \}$.

Definition (2.5): A graph G is self centered if all its vertices lie in the center C(G). Thus, the eccentric set of a self-centered graph contains only one element, that is, all the vertices have the same eccentricity.

Definition (2.6): $G^* = (V, R_1, R_2, \ldots, R_k)$ is a graph structure if $V = \bigcup_{i=1}^{k} V_i$, where each V_i is a non-empty set and $R_1, R_2, \ldots, R_k$ on V_i which are mutually disjoint such that each R_i, i = 1, 2, …., k is symmetric and irreflexive.

3. Self Center Property of Intuitionistic Fuzzy Graph Structure:

Definition (3.1): $\tilde{G} = (A, B_1, B_2, \ldots, B_k)$ is an intuitionistic fuzzy graph structure (IFGS) of a graph structure $G = (V, R_1, R_2, \ldots, R_k)$ if

$$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v) \quad \text{and} \quad \nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v) \ \forall\, u, v \in V \text{ and } i = 1, 2, \ldots, k$$

where A is an intuitionistic fuzzy subset (IFS) on V and $B_1, B_2, \ldots, B_k$ are intuitionistic fuzzy relations (IFR) on V which are mutually disjoint, symmetric and irreflexive .

Note (3.2): $\tilde{G} = (A, B_1, B_2, \ldots, B_k)$ will represent an intuitionistic fuzzy graph structure with respect to graph structure $G = (V, R_1, R_2, \ldots, R_k)$ throughout this paper.

Definition (3.3): Let $\tilde{G}$= (A, B_1, B_2,...,B_k) be a B_i - connected IFGS then B_i - length of the Path P : $v_1, v_2, \ldots, v_n$ in $\tilde{G}$ is denoted by $l_{B_i}(P) = \left(l_{\mu_{B_i}}(P),\ l_{\nu_{B_i}}(P)\right)$ where $l_{\mu_{B_i}}(P)$ and $l_{\nu_{B_i}}(P)$ are defined as $l_{\mu_{B_i}}(P) = \sum_{j=1}^{n-1} \frac{1}{\mu_{B_i}(v_j, v_{j+1})}$ and $l_{\nu_{B_i}}(P) = \sum_{j=1}^{n-1} \frac{1}{\nu_{B_i}(v_j, v_{j+1})}$.

Definition (3.4): Let $\tilde{G}$ be a B_i-connected IFGS then B_i-distance between two vertices u and v is denoted by $\delta_{B_i}(u,v) = \left(\delta_{\mu_{B_i}}(u,v),\ \delta_{\nu_{B_i}}(u,v)\right)$ where $\delta_{\mu_{B_i}}(u,v)$ and $\delta_{\nu_{B_i}}(u,v)$ are defined as

$$\delta_{\mu_{B_i}}(u,v) = \min\left\{l_{\mu_{B_i}}(P):\ \forall\ u-v \text{ path P}\right\} \text{ and } \delta_{\nu_{B_i}}(u,v) = \max\left\{l_{\nu_{B_i}}(P): \forall\ u-v \text{ path P}\right\}.$$

Definition (3.5): Let $\tilde{G}$ be a connected IFGS then distance between vertices u and v is denoted by δ(*u,v*) and is defined as

$$\delta(u,v) = \left(\min\{\delta_{\mu_{B_i}}(u,v):\ i=1,2,3,\ldots,k\},\ \max\{\delta_{\nu_{B_i}}(u,v):\ i=1,2,3,\ldots,k\}\right).$$

Definition (3.6): Let $\tilde{G}$ be a B_i-connected IFGS then, B_i– eccentricity of a vertex $u \in$ V is denoted by $e_{B_i}(u)$ = ($e_{\mu_{B_i}}(u)$, $e_{\nu_{B_i}}(u)$) where $e_{\mu_{B_i}}(u)$ and $e_{\nu_{B_i}}(u)$ are defined as

$$e_{\mu_{B_i}}(u) = \max\left\{\delta_{\mu_{B_i}}(u,v) : \forall v \in \mathrm{V}, v \neq u\right\} \text{ and } e_{\nu_{B_i}}(u) = \min\left\{\delta_{\nu_{B_i}}(u,v) : \forall v \in \mathrm{V}, v \neq u\right\}.$$

Definition(3.7): The eccentricity of a vertex *u* in IFGS $\tilde{G}$ is denoted by $e_{\tilde{G}}(u)$ and is defined as

$e_{\tilde{G}}(u)$ = (min{$e_{\mu_{B_i}}(u)$: i = 1,2,3,....., k}, max{$e_{\nu_{B_i}}(u)$: i = 1,2,3,....., k}).

Definition (3.8): Let $\tilde{G}$ be a B_i-connected IFGS then B_i -radius of $\tilde{G}$ is denoted by $r_{B_i}(\tilde{G})$ = ($r_{\mu_{B_i}}(\tilde{G})$, $r_{\nu_{B_i}}(\tilde{G})$) where $r_{\mu_{B_i}}(\tilde{G})$ and $r_{\nu_{B_i}}(\tilde{G})$ are defined as

$r_{\mu_{B_i}}(\tilde{G})$ = min{$e_{\mu_{B_i}}(u)$: ∀*u* ∈ V } and $r_{\nu_{B_i}}(\tilde{G})$ = min { $e_{\nu_{B_i}}(u)$: ∀ *u* ∈ V }.

Definition (3.9): Let $\tilde{G}$ be a connected IFGS then radius of $\tilde{G}$ is denoted by r($\tilde{G}$) and is defined as r($\tilde{G}$) = (max { $r_{\mu_{B_i}}(\tilde{G})$: i = 1,2,3,....., k}, min {$r_{\nu_{B_i}}(\tilde{G})$: i = 1,2,3,....., k}).

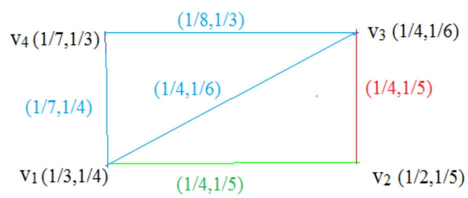
Definition (3.10): Let $\tilde{G}$ be a B_i-connected IFGS then B_i- diameter of $\tilde{G}$ is denoted by $d_{B_i}(\tilde{G})$ = ($d_{\mu_{B_i}}(u)$, $d_{\nu_{B_i}}(u)$) where $d_{\mu_{B_i}}(u)$ and $d_{\nu_{B_i}}(u)$ are defined as

$d_{\mu_{B_i}}(\tilde{G})$ = max {$e_{\mu_{B_i}}(u)$: ∀ u ∈ V } and $d_{\nu_{B_i}}(\tilde{G})$ = max { $e_{\nu_{B_i}}(u)$: ∀ u ∈ V }.

Definition (3.11): Let $\tilde{G}$ be a connected IFGS then diameter of $\tilde{G}$ is denoted by d$(\tilde{G})$ and is defined as d$(\tilde{G})$ = (min{$d_{\mu_{B_i}}(u)$: i = 1,2,3,....., k}, max{$d_{\nu_{B_i}}(u)$: i = 1,2,3,....., k}).

Example (3.12): Consider an IFGS $\tilde{G}$ = (A, B_1, B_2, B_3) such that V ={v_1, v_2, v_3, v_4}be a vertex set.

Let R_1= {(v_1, v_4), (v_1 ,v_2), (v_3, v_4)}, R_2={ (v_2, v_3)}, R_3 = {(v_1, v_2)}.



Here in this example, we can find that r$(\tilde{G})$ = (4, 5), d$(\tilde{G})$ = (4, 5), $e_{\tilde{G}}(v_k)$ = (4, 5) ∀ v_k ∈ V.

Definition (3.13): A vertex u ∈ V is called B_i - central vertex of a B_i -connected IFGS if $r_{B_i}(\tilde{G}) = e_{B_i}(u)$.

Definition (3.14): A vertex u ∈ V is called a central vertex of a connected IFGS $\tilde{G}$ if r$(\tilde{G})$ = $e_{\tilde{G}}(u)$. The set of all central vertices of a connected IFGS $\tilde{G}$ is denoted by C$(\tilde{G})$.

Definition (3.15): An intuitionistic fuzzy subgraph structure $\tilde{H}$ = (A_1, C_1, C_2,...,C_k) of an IFGS $\tilde{G}$ = (A, B_1, B_2,...,B_k) induced by the central vertices C$(\tilde{G})$ is denoted by < C $(\tilde{G})$ > and is called the Center of $\tilde{G}$.

Definition (3.16): A connected IFGS $\tilde{G}$ is self centered if every vertex of $\tilde{G}$ is a central vertex of $\tilde{G}$ i.e., r$(\tilde{G})$ = $e_{\tilde{G}}(u)$ ∀ u ∈ V. In other words $\tilde{G}$ is self centered if C$(\tilde{G})$ = V.

In the example (3.12), r$(\tilde{G})$ = $e_{\tilde{G}}(v_k)$ = (4, 5) for k = 1,2,3,4. Therefore $\tilde{G}$ is self centered.

Definition (3.17): An IFGS $\tilde{G}$ = (A,B_1,B_2,...,B_k) of G* = (V,R_1,R_2,........,R_k) is said to be complete if $\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) = \nu_A(u) \vee \nu_A(v)$ ∀ u, v ∈ V and i = 1, 2, 3,...., k.

Theorem (3.18): Every complete IFGS $\tilde{G}$ is self centered and $r_{\mu_{B_i}}(\tilde{G})$ = $\frac{1}{\mu_A(u)}$ where $\mu_A(u)$ is the least and $r_{\nu_{B_i}}(\tilde{G})$ = $\frac{1}{\nu_A(u)}$ where $\nu_A(u)$ is the greatest.

Proof: Given $\tilde{G}$ is complete intuitionistic fuzzy graph structure (CIFGS).

To prove $\tilde{G}$ is self centered. i.e. To prove every vertex is a central vertex.

For this, we claim that $\tilde{G}$ is a B_i- self central intuitionistic fuzzy graph structure and

$r_{\mu_{B_i}}(\tilde{G}) = \frac{1}{\mu_A(u)}$ where $\mu_A(u)$ is the least and $r_{\nu_{B_i}}(\tilde{G}) = \frac{1}{\nu_A(u)}$ where $\nu_A(u)$ is the greatest.

Now fix a vertex $u \in V$ such that $\mu_A(u)$ is the least membership value and $\nu_A(u)$ is the greatest vertex membership of $\tilde{G}$.

Case I - Consider all *u*-*v* path P of length (i.e. no. of edges) n in $\tilde{G}$ $\forall v \in V$.

Subcase (i): If n = 1, $l_{\mu_{B_i}}(P) = \frac{1}{\mu_A(u)}$ and $l_{\nu_{B_i}}(P) = \frac{1}{\nu_A(u)}$.

Subcase (ii): If n > 1, then one of the edges of P possesses the μ_{B_i}-strength of $\mu_A(u)$,

so that $l_{\mu_{B_i}}(P) > \frac{1}{\mu_A(u)}$ i.e., $\delta_{\mu_{B_i}}(u,v) = \min\{l_{\mu_{B_i}}(P)\} = \frac{1}{\mu_A(u)}$ $\forall v \in V$. ------(1)

Also one of the edges of P have ν_{B_i}-strength of $\nu_A(u)$, so that $l_{\mu_{B_i}}(P) > \frac{1}{\mu_A(u)}$

i.e., $\delta_{\mu_{B_i}}(u,v) = \min\{l_{\mu_{B_i}}(P)\} = \frac{1}{\mu_A(u)}$ $\forall v \in V$. ------(2)

Case II: - Let $w \neq u \in V$.

Consider all *w*-*v* path Q of l_{B_i}-length in $\tilde{G}$ $\forall v \in V$.

Subcase (i): If n = 1, $\mu_{B_i}(w, v) = \mu_A(w) \wedge \mu_A(u) \geq \mu_A(u)$ (because $\mu_A(u)$ is the least)

$\therefore l_{\mu_{B_i}}(Q) = \frac{1}{\mu_{B_i}(w,v)} \leq \frac{1}{\mu_A(u)}$

Also $\nu_{B_i}(w, v) = \nu_A(w) \vee \nu_A(u) \leq \nu_A(u)$ (because $\nu_A(u)$ is the greatest)

$\therefore l_{\nu_{B_i}}(Q) = \frac{1}{\nu_{B_i}(w,v)} \geq \frac{1}{\nu_A(u)}$

Subcase (ii): If n = 2, $\frac{1}{\mu_{B_i}(w,t)} + \frac{1}{\mu_{B_i}(t,v)} \leq \frac{1}{\mu_A(u)} + \frac{1}{\mu_A(u)} = \frac{2}{\mu_A(u)}$ (since $\mu_A(u)$ is the least)

$\therefore l_{\mu_{B_i}}(Q) \leq \frac{2}{\mu_A(u)}$

Also, $l_{\nu_{B_i}}(Q) = \frac{1}{\nu_{B_i}(w,t)} + \frac{1}{\nu_{B_i}(t,v)} \geq \frac{1}{\nu_A(u)} + \frac{1}{\nu_A(u)} = \frac{2}{\nu_A(u)}$ (since $\nu_A(u)$ is the greatest)

$\therefore l_{\nu_{B_i}}(Q) \geq \frac{2}{\nu_A(u)}$.

Subcase (iii): If $n > 2$, then $l_{\mu_{B_i}}(Q) \leq \dfrac{n}{\mu_A(u)}$ (since $\mu_A(u)$ is the least)

i.e. $\delta_{\mu_{B_i}}(w,v) = \min\{l_{\mu_{B_i}}(Q)\} \leq \dfrac{1}{\mu_A(u)}$ $\forall\, u \in V$. ------(3)

Also $l_{\nu_{B_i}}(Q) \geq \dfrac{n}{\nu_A(u)}$ (since $\nu_A(u)$ is the greatest)

i.e. $\delta_{\nu_{B_i}}(w,v) = \min\{l_{\nu_{B_i}}(Q)\} \geq \dfrac{1}{\nu_A(u)}$ $\forall\, u \in V$ ------(4)

$\therefore$ from (1) and (3), $e_{\mu_{B_i}}(u) = \max\{\delta_{\mu_{B_i}}(u,v)\} = \dfrac{1}{\mu_A(u)}$ $\forall\, u \in V$. ------(5)

And from (2) and (4), $e_{\nu_{B_i}}(u) = \max\{\delta_{\nu_{B_i}}(u,v)\} = \dfrac{1}{\nu_A(u)}$ $\forall\, u \in V$ ------(6)

$\Rightarrow \tilde{G}$ is a B_i- self central intuitionistic fuzzy graph structure.

Now, $r_{\mu_{B_i}}(\tilde{G}) = \min\{e_{\mu_{B_i}}(u)\} = \dfrac{1}{\mu_A(u)}$ (by equation (3))

$\therefore$ $r_{\mu_{B_i}}(\tilde{G}) = \dfrac{1}{\mu_A(u)}$ where $\mu_A(u)$ is the least.

Also $r_{\nu_{B_i}}(\tilde{G}) = \min\{e_{\nu_{B_i}}(u)\} = \dfrac{1}{\nu_A(u)}$ (by equation (6))

$\therefore$ $r_{\nu_{B_i}}(\tilde{G}) = \dfrac{1}{\nu_A(u)}$ where $\nu_A(u)$ is the greatest.

$\therefore$ from (5) and (6), every vertex of $\tilde{G}$ is a central vertex.

Hence $\tilde{G}$ is a self centered intuitionistic fuzzy graph structure.

Corollary (3.19): Every complete intuitionistic fuzzy graph structure $\tilde{G}$ is self centered and

$r(\tilde{G}) = \left(\dfrac{1}{\mu_A(u)}, \dfrac{1}{\nu_A(u)}\right)$ where $\mu_A(u)$ is the least and $\nu_A(u)$ is the greatest.

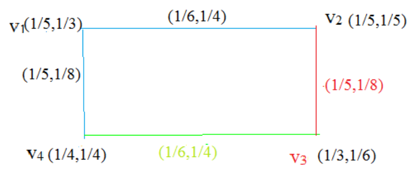
Proof: $r(\tilde{G}) = (\min\{r_{\mu_{B_i}}(\tilde{G}) : i = 1,2,3 \ldots\ldots, k\}, \min\{r_{\nu_{B_i}}(\tilde{G}) : i = 1,2,3 \ldots\ldots, k\})$

$r(\tilde{G}) = \left(\dfrac{1}{\mu_A(u)}, \dfrac{1}{\nu_A(u)}\right)$ (since $\mu_A(u)$ is the least and $\nu_A(u)$ is the greatest)

Remark (3.20): Converse of the above theorem is not true.

Example (3.21): Consider intuitionistic fuzzy graph structure $\tilde{G} = (A, B_1, B_2, B_3)$ such that

V = { v_1, v_2, v_3, v_4} is a vertex set.



Here r($\tilde{G}$) = e (v) $\forall$ $v_j \in V$. j= 1,2,3,4

$\therefore$ $\tilde{G}$ is self centered, but $\tilde{G}$ is not complete intuitionistic fuzzy graph structure.

Definition (3.22): Let $\tilde{G}$ be an IFGS then μ_{B_i} - strength (or ν_{B_i} -strength) of the paths connecting two vertices *u,v* is defined as max{$S_{\mu_{B_i}}$ }(or max{$S_{\nu_{B_i}}$ }) and is denoted by $\mu_{B_i}{}^{\infty}$(*u, v*) (or $\nu_{B_i}{}^{\infty}$(*u, v*)).

Definition (3.23): The strength of the paths connecting two vertices *u, v* is defined as max { S_{B_i} } & is denoted by μ^{∞} (*u, v*).

Note(3.24): If the same edge posses both the μ_{B_i} -strength and ν_{B_i} -strength values then it is the strength of the strongest path P and is denoted as $S_{B_i} = (\mu_{B_i}{}^{\infty}(u,v).\nu_{B_i}{}^{\infty}(u,v))$ for all i =1,2,3 ...., k.

Theorem (3.25): If $\tilde{G}$ is then for atleast one edge, $\mu_{B_i}{}^{\infty}$ *(u, v) =* μ_{B_i} *(u, v) and* $\nu_{B_i}{}^{\infty}$ *(u, v) =*ν_{B_i} *(u, v).*

Proof: Given $\tilde{G}$ is a complete intuitionistic fuzzy graph structure.

Consider a vertex *u* with membership value μ_A(*u*) and non-membership value ν_A(*u*)

Case I : Let μ_A(*u*) be the least and ν_A(*u*) the greatest $\forall$ *u* $\in$V.

Let *u, v*$\in$V then (μ_{B_i} (*u, v*),ν_{B_i} (*u, v*)) = (μ_A(*u*), ν_A(*u*)) and ($\mu_{B_i}{}^{\infty}$ *(u, v),* $\nu_{B_i}{}^{\infty}$ *(u, v)) = (* μ_A*(u),* ν_A(*u*)).

The strength of all the edges which are incident on the vertex *u* is (μ_A(*u*), ν_A(*u*)).

Case II : Let μ_A(*u*) be the least and ν_A(*w*) the greatest where *u* $\neq$ *w*

$\therefore$ (μ_{B_i} *(u,w),* ν_{B_i} *(u,w)) =* (μ_A(*u*), ν_A(*w*)).

Since $\tilde{G}$ is complete IFGS, therefore there will be an edge between *u* and *w*

$\therefore$ $\mu_{B_i}{}^{\infty}$ *(u,w) =* μ_A*(u)* and $\nu_{B_i}{}^{\infty}$ *(u,w) =* ν_A*(w)*

Lemma (3.26): An intuitionistic fuzzy graph structure $\tilde{G}$ is self centered if and only if $r_{\mu_{B_i}}$ ($\tilde{G}$) = $d_{\mu_{B_i}}$ ($\tilde{G}$) and $r_{\nu_{B_i}}$ ($\tilde{G}$) = $d_{\nu_{B_i}}$ ($\tilde{G}$) i.e. if and only if r_{B_i} ($\tilde{G}$) = d_{μ_i} ($\tilde{G}$).

Proof: It follows from the above definitions.

Lemma (3.27): An IFGS $\tilde{G}$ is self centered if and only if $r(\tilde{G}) = d(\tilde{G})$.

In example (3.21), $r(\tilde{G}) = d(\tilde{G})$ $\forall v_j \in V$. j = 1,2,3,4 $\therefore \tilde{G}$ is self centered,

Theorem (3.28): (Embedding Theorem): Let $\widetilde{H}$ = (A, B'_1, B'_2,...,B'_k) be a connected, B_i-self centered intuitionistic fuzzy graph structure then there exists a connected intuitionistic fuzzy graph structure $\tilde{G}$ such that $< C(\tilde{G}) >$ is isomorphic to $\widetilde{H}$. Also $d_{\mu_{B_i}}(\tilde{G}) = 2\, r_{\mu_{B_i}}(\tilde{G})$ and $d_{\nu_{B_i}}(\tilde{G}) = 2\, r_{\nu_{B_i}}(\tilde{G})$.

Proof: Given $\widetilde{H}$ is connected B_i -self centered intuitionistic fuzzy graph structure.

Let $d_{\mu_{B_i}}(\widetilde{H})$ = p and $d_{\nu_{B_i}}(\widetilde{H})$ = q.

Then construct $\tilde{G}$ = (A,B_1,B_2,....,B_k) from $\widetilde{H}$ as follows:

Take two vertices *u,v* ∈ V with $\mu_A(u) = \mu_A(v) = \frac{1}{p}$, $\nu_A(u) = \nu_A(v) = \frac{1}{q}$ and join all the vertices of $\widetilde{H}$ to both *u* & *v* with $\mu_{B_i}(u, w) = \mu_{B_i}(v, w) = \frac{1}{p}$, $\nu_{B_i}(u, w) = \nu_{B_i}(v, w) = \frac{1}{q}$ $\forall w \in V'$.

Put $\mu_A = \mu_A'$, $\nu_A = \nu_A'$ for all vertices in $\widetilde{H}$ and $\mu_{B_i} = \mu_{B_i}'$, $\nu_{B_i} = \nu_{B_i}'$ for all edges in $\widetilde{H}$.

Claim that $\tilde{G}$ is a IFGS.

First note that $\mu_A(u) \leq \mu_A(w)$ $\forall w \in \widetilde{H}$.

If possible, let $\mu_A(u) > \mu_A(w)$ for atleast one vertex $w \in \widetilde{H}$,

Then $\frac{1}{p} > \mu_A(u) \Rightarrow p < \frac{1}{\mu_A(w)} \leq \frac{1}{\mu_{B_i}(u,w)}$

$\Rightarrow$ the last inequality holds for every $w \in V'$ (since $\widetilde{H}$ is a IFGS)

$\Rightarrow \frac{1}{\mu_{B_i}(u,w)} > p$ $\forall w \in \widetilde{H}$ which contradicts that $d_{\mu_{B_i}}(\widetilde{H})$ = p

$\therefore \mu_{B_i}(u) \leq \mu_{B_i}(w)$ $\forall w \in V'$.

And $\mu_{B_i}(u, w) \leq min\{\mu_A(u), \mu_A(w)\} = \frac{1}{p}$ $\forall w \in V'$

Similarly $\nu_{B_i}(u, w) \leq min\{\nu_A(u), \nu_A(w)\} = \frac{1}{p} \forall w \in V'$

Note that $\nu_A(u) \leq \nu_A(w)$, $\nu_A(v) \leq \nu_A(w)$ $\forall w \in V'$ (as $d_{\nu_{B_i}}(\widetilde{H})$ = q)

$\nu_{B_i}(u, w) \leq \max\{\nu_A(u), \nu_A(w)\} = \dfrac{1}{2q}$.

Thus $\tilde{G}$ is an IFGS.

Also $e_{\mu_{B_i}}(w) = p \quad \forall\, w \in V'$, $e_{\mu_{B_i}}(u) = e_{\mu_{B_i}}(v) = \dfrac{1}{\mu_{B_i}(u, w)} + \dfrac{1}{\mu_{B_i}(w, v)} = 2p$

$\therefore\ r_{\mu_{B_i}}(\tilde{G}) = p$ and $d_{\mu_{B_i}}(\tilde{G}) = 2p$.

Similarly, $e_{\nu_{B_i}}(w) = q \quad \forall\, w \in V'$, $e_{\nu_{B_i}}(u) = e_{\nu_{B_i}}(v) = \dfrac{1}{\nu_{B_i}(u, w)} = 2q$

$\therefore\ r_{\nu_{B_i}}(\tilde{G}) = q$ and $d_{\nu_{B_i}}(\tilde{G}) = 2q$

$\Rightarrow < C(\tilde{G}) >$ is isomorphic to $\widetilde{H}$.

The following example shows that $< C(\tilde{G}) >$ is isomorphic to $\widetilde{H}$ and also

$d_{\mu_{B_i}}(\tilde{G}) = 2\, r_{\mu_{B_i}}(\tilde{G})$, and $d_{\nu_{B_i}}(\tilde{G}) = 2\, r_{\nu_{B_i}}(\tilde{G})$.

Theorem (3.29): The necessary and sufficient condition for an IFGS $\tilde{G}$ to be self centered is that $\delta_{B_i}(u,v) \leq r_{B_i}(\tilde{G})$ $\forall\, u,v \in V$.

Proof: Necessary Condition: Let $\tilde{G}$ be self centered IFGS.

i.e. $e_{B_i}(u) = e_{B_i}(v) \quad \forall\, u,v \in V$ and $r_{B_i}(\tilde{G}) = e_{B_i}(u) \quad \forall\, u \in V$.

Now to prove that $\delta_{B_i}(u,v) \leq r_{B_i}(\tilde{G}) \quad \forall\, u,v \in V$ ----(1)

This is possible only when $e_{B_i}(u) = r_{B_i}(\tilde{G}) \quad \forall\, u,v \in V$.

Since G is self centered intuitionistic fuzzy graph structure, the above inequality becomes

$\delta_{B_i}(u,v) \leq r_{B_i}(\tilde{G})$.

Sufficient Condition: Given $\delta_{B_i}(u,v) \leq r_{B_i}(\tilde{G})\ \forall\, u,v \in V$.

To prove $\tilde{G}$ is a self centered intuitionistic fuzzy graph structure.

Suppose $\tilde{G}$ is not self centered intuitionistic fuzzy graph structure.

$\therefore\ e_{B_i}(u) \neq r_{B_i}(\tilde{G})$ for some $u \in V$.

Assume that $e_{B_i}(u)$ is the least value among all other eccentricity.

i.e. $r_{B_i}(\tilde{G}) = e_{B_i}(u) \quad \forall\ u \in V$ ----(2) where $e_{B_i}(u) < e_{B_i}(v)$ for some $u,v \in V$.

and $\delta_{B_i}(u,v) = e_{B_i}(v) > e_{B_i}(u)$ for some $u,v \in V$.

$\Rightarrow \delta_{B_i}(u,v) > e_{B_i}(u)$ for some $u,v \in V$.

$\Rightarrow \delta_{B_i}(u,v) > r_{B_i}(u)$ for some $u,v \in V$ (by using (2))

which is a contradiction to the fact that $\delta_{B_i}(u,v) \le r_{B_i}(\tilde{G})\ \forall\ u,v \in V$.

$\Rightarrow$ Our supposition is wrong.

$\therefore$ $\tilde{G}$ is a self centered intuitionistic fuzzy graph structure.

Definition (3.30): An IFGS $\tilde{G}$ is said to be bipartite if vertex set V can be partitioned into two non - empty sets U_1 and U_2 such that

(i) $\mu_{B_i}(u, v) = 0$ and $\nu_{B_i}(u, v) = 0$ if $u, v \in U_1$ or $u, v \in U_2$

(ii) $\mu_{B_i}(u, v) > 0$ and $\nu_{B_i}(u, v) > 0$ if $u \in U_1$ or $v \in U_2$,

Or

(i) $\mu_{B_i}(u, v) = 0$ and $\nu_{B_i}(u, v) > 0$ if $u \in U_1$ or $v \in U_2$

(ii) $\mu_{B_i}(u, v) > 0$ and $\nu_{B_i}(u, v) = 0$ if $u \in U_1$ or $v \in U_2$

Definition (3.31): A bipartite IFGS $\tilde{G}$ is said to be complete if $\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) = \nu_A(u) \vee \nu_A(v)$, $\forall\ u \in U_1$ & $v \in U_2$ and i = 1,2,3,......,k. It is denoted by K_{U_1,U_2}

Theorem (3.32): Let $\tilde{G}$ be an intuitionistic fuzzy graph structure. If $\tilde{G}$ is a complete bipartite intuitionistic fuzzy graph structure then complement of $\tilde{G}$ is a self centered intuitionistic fuzzy graph structure.

Proof: A bipartite intuitionistic fuzzy graph structure $\tilde{G}$ = (A, $B_1, B_2, \ldots, B_k$) is said to be complete if $\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$, $\nu_{B_i}(u, v) = \nu_A(u) \vee \nu_A(v)$ $\forall\ u \in U_1$ & $v \in U_2$ and i = 1,2,3,......,k

And $\mu_A(u,v) = 0$, $\nu_A(u,v) = 0 \quad \forall\ u, v \in U_1$ or $u, v \in U_2$ -----(1)

Now $\overline{\mu_{B_i}}(u, v) = \mu_A(u) \wedge \mu_A(v) - \mu_{B_i}(u,v)$, $\overline{\nu_{B_i}}(u,v) = \nu_A(u) \vee \nu_A(v) - \nu_{B_i}(u, v)$ ----(2)

By using eq(1), $\overline{\mu_{B_i}}(u,v) = \mu_A(u) \wedge \mu_A(v)$, $\overline{\nu_{B_i}}(u,v) = \nu_A(u) \vee \nu_A(v)$ $\forall\ u,v \in U_1$ or $u,v \in U_2$ --- (3)

Hence from eq, (1), (2), (3), the complement of $\tilde{G}$ has two components and each component is complete intuitionistic fuzzy graph structure which is a self centered intuitionistic fuzzy graph structure by theorem (3.22). Hence the result is proved.

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